MAT 121 §3.6 #s 7, 16, 17, 23, 26, 32, 33, 40

#s 7-16 Info is given about a polynomial <u>whose coefficients are real numbers</u>. Find the remaining zeros of f.

(7) f(x) is degree 3 zeros: 3, $4-i$

$\Rightarrow$ $4+i$ is ALSO a zero by Conjugate Pairs Thm.

$\Rightarrow$ $\boxed{f(x) = (x-3)(x-(4-i))(x-(4+i))}$ FULL CREDIT ON TEST

$$= (x-3)\left[x^2 - (4+i)x - (4-i)x + (4-i)(4+i)\right]$$

$$= (x-3)\left[x^2 - 4x - ix - 4x + ix + 4^2 + 1^2\right]$$

$$= (x-3)\left[x^2 - 4x - 4x + 17\right]$$

$$= (x-3)\left[x^2 - 8x + 17\right]$$

$$= x^3 - 8x^2 + 17x - 3x^2 + 24x - 51$$

$$= \boxed{x^3 - 11x^2 + 41x - 51 = f(x)}$$ $\rightarrow$ Bonus on Test.

(16) Degree 6; zeros i, $3-2i$, $-2+i$ FULL CREDIT

$$f(x) = (x-i)(x+i)(x-(3-2i))(x-(3+2i))(x-(-2+i))(x-(-2-i))$$

$$= (x^2+1)(x^2-6x+13)(x^2+4x+5)$$

$$= (x^2+1)(x^4+4x^3+5x^2-6x^3-24x^2-30x+13x^2+52x+65)$$

$$= (x^2+1)(x^4-2x^3-6x^2+22x+65)$$

$$= x^6 - 2x^5 - 6x^4 + 22x^3 + 65x^2 + x^4 - 2x^3 - 6x^2 + 22x + 65.$$

$$= x^6 - 2x^5 - 5x^4 + 20x^3 + 59x^2 + 22x + 65$$ BONUS

OOPS! #s 7-16 I didn't follow instructions. All they wanted was a listing of the zeros.

#7: $x = 3, 4 \pm i$ are zeros.

#16: $x = \pm i, 3 \pm 2i, -2 \pm i$ are zeros.

Still, good exercise on building factored form from the zeros and then expanding

Bonus on Tests.

#s 17-22 Form a polynomial $f(x)$ with real coefficients having the given degree and zeros.

17 Degree 4; zeros $3+2i$; 4, multiplicity 2

$f(x) = (x-4)^2(x-(3+2i))(x-(3-2i))$

$= (x^2-8x+16)(x^2-6x+13)$

$= x^4 - 6x^3 + 13x^2 - 8x^3 + 48x^2 - 104x + 16x^2 - 96x + 208$

$= x^4 - 14x^3 + 77x^2 - 200x + 208$

#s 23-30: Use the given zero to find the remaining zeros of $f(x)$.

MAT 121 §3.6 #s 23, 26, 32, 33, 40

23 $f(x) = x^3 - 4x^2 + 4x - 16$; zero: $2i$

$$
\begin{array}{r|rrrr}
2i & 1 & -4 & 4 & -16 \\
 & & 2i & -4-8i & 16 \\
\hline
-2i & 1 & -4+2i & -8i & 0 \\
 & & -2i & 8i & r \\
\hline
x-4 \rightsquigarrow & 1 & -4 & 0 &
\end{array}
$$

$2i(-4+2i)$
$= -8i + 4i^2$
$= -8i - 4$

$(2i)(-8i)$
$= 16$

This gives $(x-2i)(x+2i)(x-4)$

$\boxed{\text{Zeros are } x = 4, \pm 2i.}$

Another way to do this one (Disobeying Instructions!) for good factoring skills:

$x^3 - 4x^2 + 4x - 16$
$= x^2(x-4) + 4(x-4)$
$= (x-4)(x^2+4)$

So, $x = 4$ & from $x^2 + 4 = 0$, we obtain $x = \pm 2i$, as well.

<u>But be sure to work these with the specified techniques on tests.</u> Alternate ways are fine for checking (Confidence), so I MIGHT give questions on the test that can be done more easily, just to cut down some of the difficulties in doing it <u>the specified way.</u>

MAT 121 §3.6 #s 26, 32, 33, 40

26 $h(x) = 3x^4 + 5x^3 + 25x^2 + 45x - 18$; zero: $3i$

$$
\begin{array}{r|rrrrr}
3i & 3 & 5 & 25 & 45 & -18 \\
 & & 9i & -27+15i & -45-6i & +18 \\
\hline
-3i & 3 & 5+9i & -2+15i & -6i & \\
 & & -9i & -15i & 6i & \\
\hline
 & 3 & 5 & -2 & 0 &
\end{array}
$$

$(3i)(5+9i) = 15i - 27$

$(3i)(-2+15i) = -6i - 45$

$x = \pm 3i$ so far. Now, solve $3x^2 + 5x - 2 = 0$

$\Longrightarrow (3x-1)(x+2) = 0$

$\Longrightarrow x = \frac{1}{3}, -2$

$\boxed{\text{FINAL ANS: zeros are } -2, \tfrac{1}{3}, \pm 3i}$

#s 31-40 Find the complex zeros of each polynomial function. Write f in factored form.

32 $f(x) = x^4 - 1 = (x^2 - 1)(x^2 + 1) =$

$= (x-1)(x+1)(x-i)(x+i)$

zeros: $x = \pm 1, \pm i$

MAT 121 §3.6 #s 33, 40

33) $f(x) = x^3 - 8x^2 + 25x - 26$ 3 or 1 positive zeros

$f(-x) = -x^3 - 8x^2 - 25x - 26$ ZERO negative zeros

Rat'l zeros: $\pm 1, \pm 2, \pm 13, \pm 26$

$$\begin{array}{r|rrrr} 1 & 1 & -8 & 25 & -26 \\ & & 1 & -7 & 18 \\ \hline & 1 & -7 & 18 & \text{No} \end{array} \qquad \begin{array}{r|rrrr} 2 & 1 & -8 & 25 & -26 \\ & & 2 & -12 & 26 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

Yes!

So $f(x) = (x-2)(x^2 - 6x + 13)$. Now,

break down $x^2 - 6x + 13 \overset{SET}{=} 0$

$$x^2 - 6x + 3^2 = -13 + 3^2$$

$$(x-3)^2 = -4$$

$$x - 3 = \pm\sqrt{-4} = \pm 2i$$

$$x = 3 \pm 2i$$

zeros: $x = 2, 3 \pm 2i$

Factored Form: $f(x) = (x-2)(x-(3+2i))(x-(3-2i))$

40) $f(x) = 2x^4 + x^3 - 35x^2 - 113x + 65$

2 or 0 positive roots

$f(-x) = 2x^4 - x^3 - 35x^2 + 113x + 65$

2 or 0 negative zeros.

$$\begin{array}{r|l} 5 & 65 \\ \hline & 13 \end{array}$$

Rat'l zeros:

$\pm 1, \pm \frac{1}{2}, \pm 5, \pm \frac{5}{2}, \pm 13, \pm \frac{13}{2}, \pm 65, \pm \frac{65}{2}$

$$\begin{array}{r|rrrrr} 1 & 2 & 1 & -35 & -113 & 65 \\ & & 2 & 3 & -32 & \\ \hline & 2 & 3 & -32 & \text{No} & \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 2 & 1 & -35 & -113 & 65 \\ & & -2 & +1 & 34 & \text{No} \\ \hline & 2 & -1 & -34 & 9 & \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 2 & 1 & -35 & -113 & 65 \\ & & 4 & 8 & -81 & \\ \hline & 2 & 5 & -27 & \text{No} & \end{array}$$

2 isn't even a candidate!

$$\begin{array}{r|rrrrr} -2 & 2 & 1 & -35 & -113 & 65 \\ & & -4 & 6 & & \\ \hline & 2 & -3 & -29 & & \end{array}$$

$$\begin{array}{r|rrrrr} 5 & 2 & 1 & -35 & -113 & 65 \\ & & 10 & 55 & 100 & -65 \\ \hline & 2 & 11 & 20 & -13 & 0 \end{array}$$

Yes!

x=5 is a zero

$$\begin{array}{r|rr} 13 & 2 & 11 \\ & & 26 \text{ No} \\ \hline & 2 & 37 \end{array}$$

$$\begin{array}{r|rrrr} -13 & 2 & 11 & 20 & -13 \\ & & -26 & \text{HUGE} & \\ \hline & 2 & -15 & \text{No} & \end{array}$$

NOTE: ONCE WE SPLIT OFF THE $(x-5)$, WE'RE NOW WORKING WITH $2x^3+11x^2+20x-13$ instead of original.

MAT 121 §3.6 #40

Working on $2x^3+11x^2+20x-13$

It looks like $\pm 13, \pm 65$ will be too big.

And ± 65 is out because the new $a_n = 2$ & $a_0 = -13$.

So let's look again @ the fractions we've avoided:

$\pm \frac{1}{2}, \pm \frac{13}{2}$ is it.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 11 & 20 & -13 \\ & & 1 & 6 & 13 \\ \hline & 2 & 12 & 26 & 0 \end{array}$$

Sweet!

$x = \frac{1}{2}$ is a zero

Now it's down to $2x^2+12x+26 \overset{SET}{=} 0$

$2(x^2+6x+13)=0$

$x^2+6x+13=0$

$x^2+6x = -13$

$x^2+6x+3^2 = -13+9$

$(x+3)^2 = -4$

$x+3 = \pm 2i$

$x = -3 \pm 2i$

Complex zeros: $x = \frac{1}{2}, 5, -3 \pm 2i$

So $f(x) = 2x^4 + x^3 - 35x^2 - 113x + 65$

$= 2\left(x-\frac{1}{2}\right)(x-5)(x-(-3+2i))(x-(-3-2i))$

Don't forget the leading coefficient